DELAY TIME MEASUREMENTS OF THE PROPAGATION OF RADIO WAVES IN THE ATMOSPHERE

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Between 1964 and 1970 the U.S. Army Corps of Engineers established networks of accurate positions around the world. The positions were determined by means of the Geodetic Secor System. This system consists of four ground stations and a satellite. The ground stations transmit radio signals to the satellite, and the signals are then returned to the ground stations. The time between transmission and reception of the signal at the ground station is measured and used to calculate the distance between ground stations and satellite. In order to convert the time measurement into a distance it is necessary to know the wave velocity along the path of which the wave is traveling. The wave velocity depends upon the medium in which the wave is traveling.and is characterized by the refractive index. The refractive index of a medium is the ratio of the wave velocity in the vacuum to the wave velocity in medium. For all material media, the refractive index is a function of the wave frequency and some physical properties of the specific medium. The frequency dependency of the refractive index is called dispersion. The propagation of an electromagnetic wave in a dispersive medium is characterized by the phase refractive index and the group refractive index. The phase refractive index is derived from the velocity with which the phase (the crest) of a wave propagates; the group refractive index is derived from the velocity with which the energy or the signal of the wave propagates. A thorough analysis of the wave propagation in material media leads to additional concepts of the refractive index. An excellent book on this subject is *Wave Propagation and Group Velocity* by Leon Brillouin. For our considerations, however, the concepts of phase velocity and group velocity are sufficient.

The group refractive index n_g and the phase refractive index n_p are related to each other by Equation (1).

$$n_g = n_p + f_c \frac{dn_p}{df_c} \tag{1}$$

The carrier frequency in this equation is f_c. Appleton and Hartree developed the expression for the phase refractive index of a magnetoionic medium. The index is a complex function and depends on the frequency of the wave, the electron density, the magnetic field of the earth, and the collision frequency of electrons with ions and molecules. It can be shown that for frequencies above 200 MHz, the Appleton-Hartree equation can be simplified so that the group refractive index can be expressed as shown in Equation (2).

$$n_g = 1 + 40.365 \frac{N}{f_c^2} \qquad (2)$$

N is the electron density. The errors resulting from this simplification are few parts in a million and are decreasing with increasing frequency.

The traveling time T of a wave between two points is given by Equation (3).

$$T = \frac{1}{c_0} \int n_g \, dr \qquad (3)$$

The integral has to be taken along the wave path between the two points. If the simplified group refractive index of the ionosphere is inserted into Equation (3), the integral is split into two terms as shown in Equation (4).

$$T = \frac{r}{c_0} + \frac{40.365}{c_0 f_c^2} \int N \, dr = T_0 + \tau \qquad (4)$$

The term r/c_0 represents the traveling time if the medium between the two points were vacuum. The second term represents the additional traveling time which is caused by the presence of the ionosphere. This term is called the ionospheric delay time because of the delaying influence of the ionosphere on the signal.* It is important to note that the phase refractive index of the ionosphere is smaller than one and, therefore the phase of the wave travels faster in the ionosphere than in vacuum.

The SECOR signals are transmitted on a carrier f_c from the ground stations to the satellite and transmitted back from the satellite to the ground stations on two different carriers, αf_c and βf_c.. The ratio of α to β is two to one. The round-trip time of the signal using the α carrier is T_α, and the round-trip time of the signal using the β carrier is T_β. Both times are measured by the SECOR system and subsequently subtracted from each other. The time difference ΔT is given by Equation (5) which relates the total electron content to the measured time difference and known parameters. The ionospheric delay

$$\Delta T = T_\beta - T_\alpha = \frac{40.365}{c_0 f_c^2} \left(\frac{1}{\beta^2} - \frac{1}{\alpha^2} \right) \int N \, dr \qquad (5)$$

time for the α downlink can now be calculated as shown in Equation (6).

$$\tau_\alpha = \frac{\Delta T}{\alpha^2/\beta^2 - 1} \qquad (6)$$

The precision of the ionospheric measurements was determined by a collocation experiment. Two SECOR stations were placed at a distance of 257 meters on an installation at Herndon, Virginia, and were used to track the same satellite. Because the two stations were so close

*The integral in equation (4) represents the total electron content along the wave path.

together the signals from each station travel through the same ionospheric region on their way to and from the satellite. The ionospheric delay times measured by the stations should therefore be identical. Figure 1 shows the results from two tracks of the collocation experiment. The satellite used was EGRS-13, a satellite in an orbital altitude of about 1100 kilometers and near 90° inclination. The table contains the ranges between satellite and ground stations and the ionospheric correction. The process of determining the distance between satellite and ground sattion is as follows: The round-trip time of the signal using the alpha-carrier is measured. This time is multiplied by the vacuum velocity of light and divided by two which yields a distance that is too large by the ionospheric correction. The true distance is obtained by subtracting the ionospheric correction from the vacuum distance. The ionospheric correction is obtained by dividing the alpha-carrier delay time by 3.118 (the delay time is measured in nanoseconds and the ionospheric correction is measured in meters). To obtain the total electron content in electrons per square meter, the ionospheric correction has to be multiplied by 4.673×10^{15}. One can see that the IC for stations 0 and 4 are not identical. Figure 2 shows the difference of the ionospheric correction of one of the preceding tracks and the deviation of the mean. The collocation experiment has shown that the standard deviation of the measured ionospheric correction is about ±3 meters. Figure 3 shows the results of three tracks of the collocation experiment. The abscissa is the elevation angle and the ordinate is the delay time for a 1600-MHz carrier. Please take note of the characteristic shape of curves 777 and 797. The points where the curves reverse is at the maximum elevation angle.

In Figure 4 you see the first geodetic network which was established by SECOR, which provides a tie between Japan and Hawaii. The orbital altitudes of the satellites used in this program were about 900 kilometers. The program was started in August 1964 and completed in July 1966. Figure 5 shows the SECOR equatorial belt network. Satellites of about 3800 kilometers altitude were used because of the large distance between stations. The program was initiated in July 1966 and completed in April 1970. Figure 6 shows the results of a track taken at the island of Woleai. In the graph at the lower left the ionospheric correction is plotted versus time. The satellite is EGRS-3, which has an orbital altitude of about 925 kilometers. The dotted line in the lower left graph is teh ionospheric correction multiplied by satellite altitude over range. This calculation was performed to obtain some extrapolated values for the total electron content above the station. The graph in the upper left of the figure shows the range to the satellite and the elevation angle of the range (dotted line) as a function of time. The right side of the figure shows the suborbital plot of the satellite orbit. At the center of the graph is the location of the station. This track was observed at 6:30 a.m. One can see that at this time the ionospheric correction is small, around five meters. Figure 7 shows similar plots. The track has been observed about noon. At this time the ionospheric correction is much larger and varies from about 90 meters to 180 meters. Figure 8 shows a four-station track in the Pacific Ocean area. Stations were located at Truk, Swallow, Kusaii, and Gizo. In this graph the delay time for a 1600-MHz carrier is plotted against the elevation angle. The local time of the track is about 9:00 p.m. The curves appear to be fairly well correllated. Figure 9 shows a similar set of curves, but taken about 6 a.m., local time.

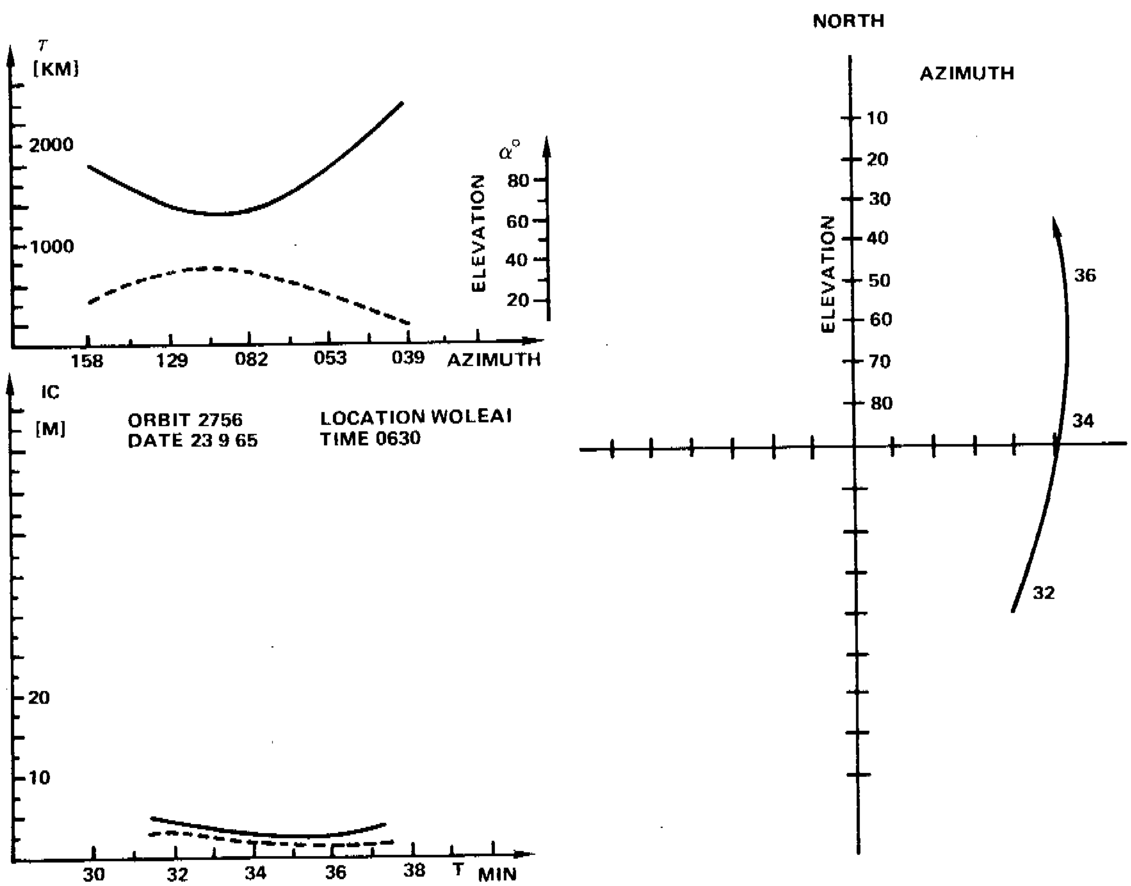

Figure 6. Results of EGRS-3 taken at the island of Woleai.

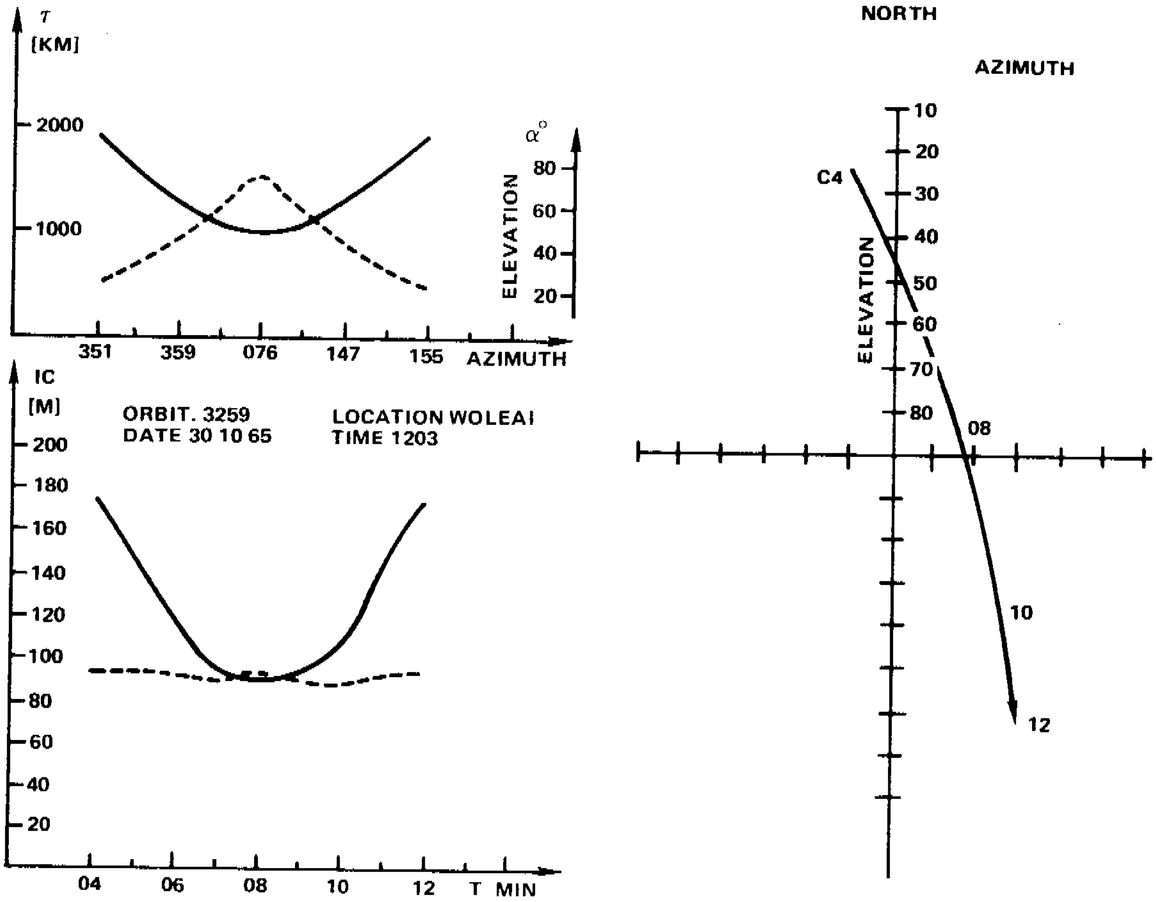

Figure 7. Results of tracks similar to those in Figure 6.

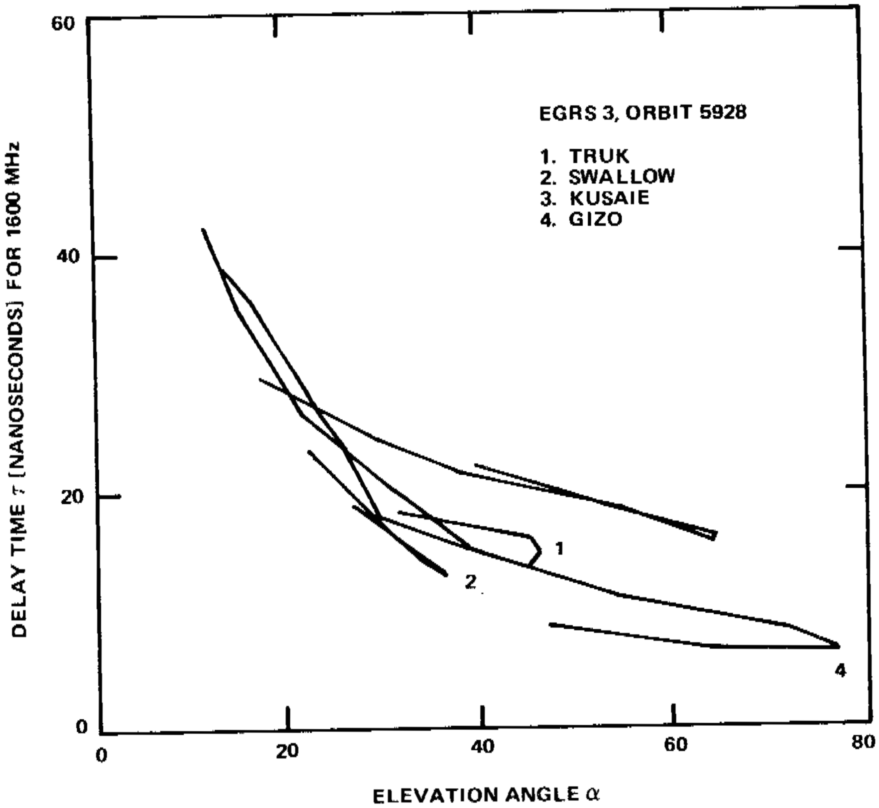

Figure 8. Results of a four-station track in the Pacific Ocean area.

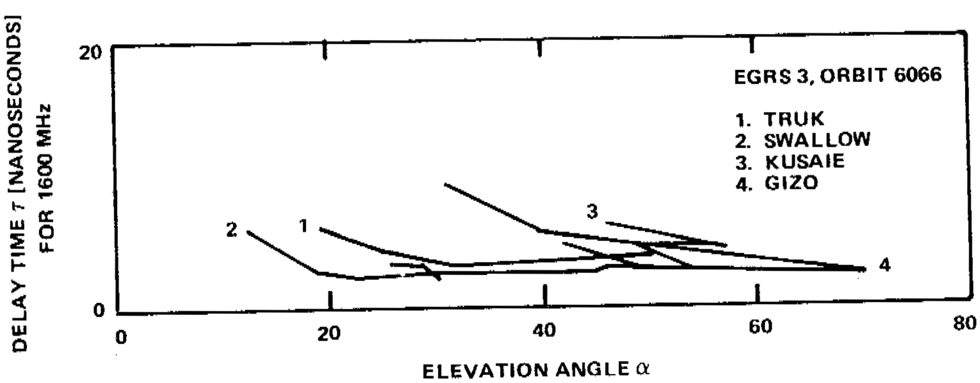

Figure 9. Curves similar to those in Figure 8.

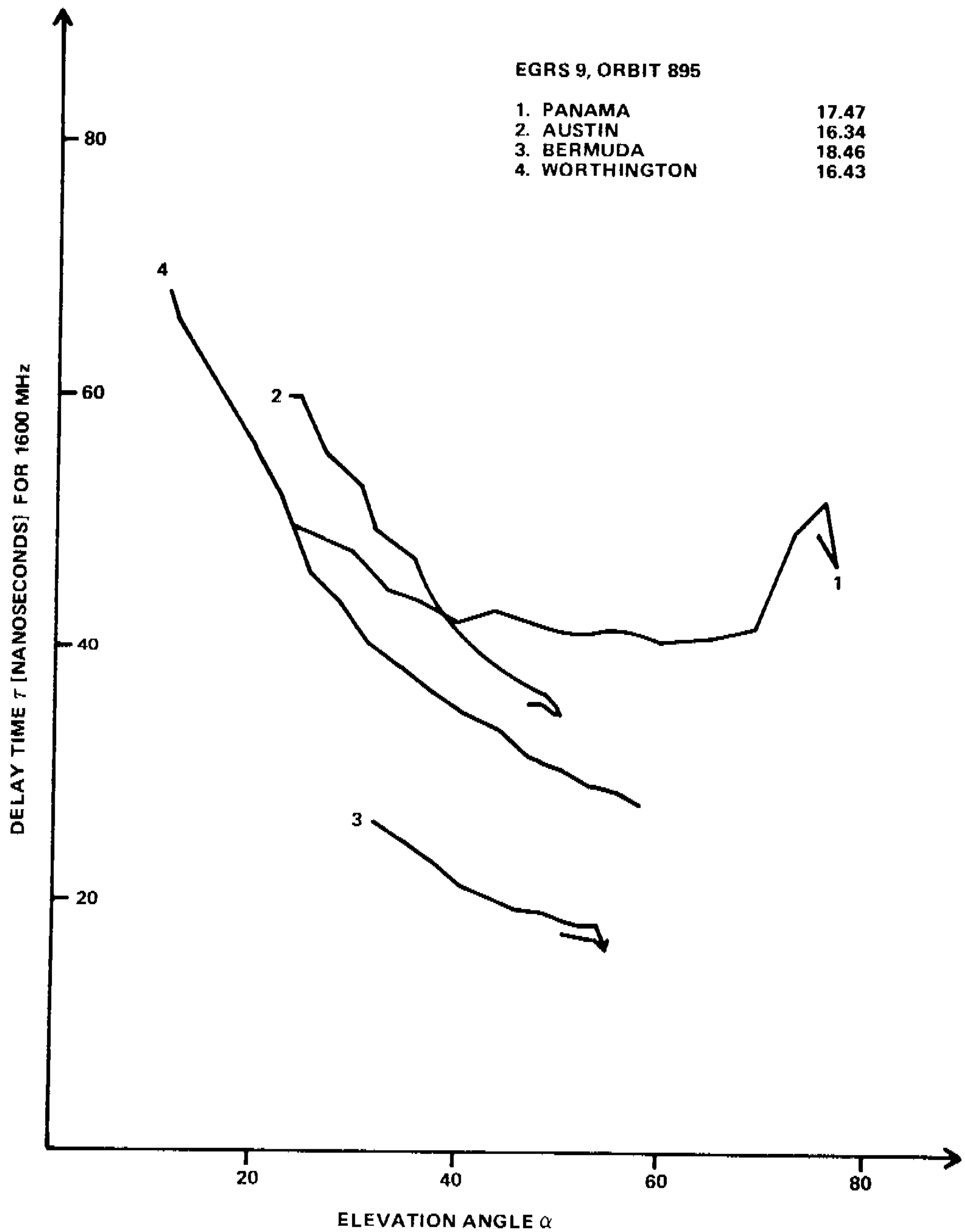

Figure 10. Ionospheric delay times measured at a four-station track during the equatorial belt operation.

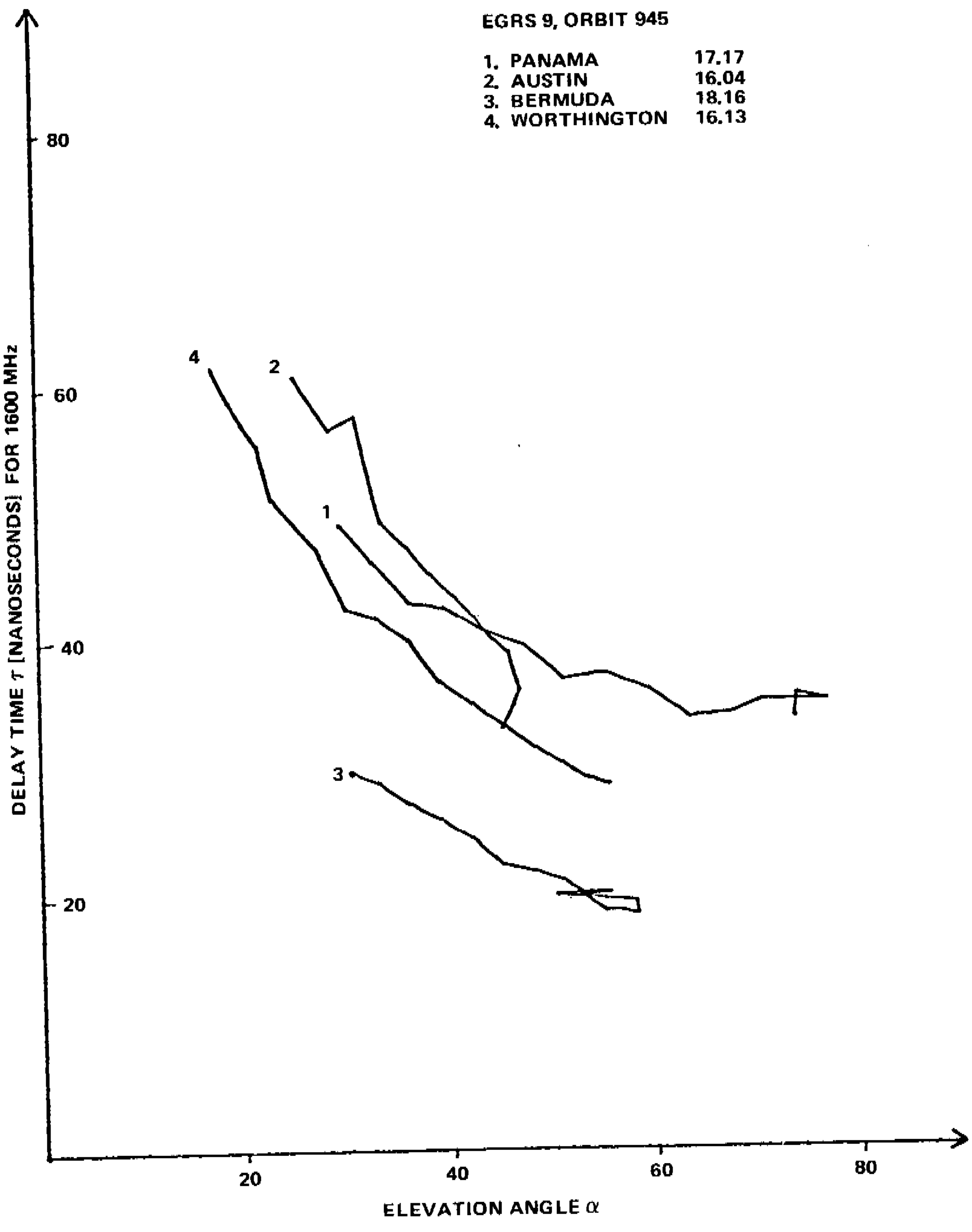

Figure 11. Results of a track taken at the same stations as those used for Figure 10, but a few days later.

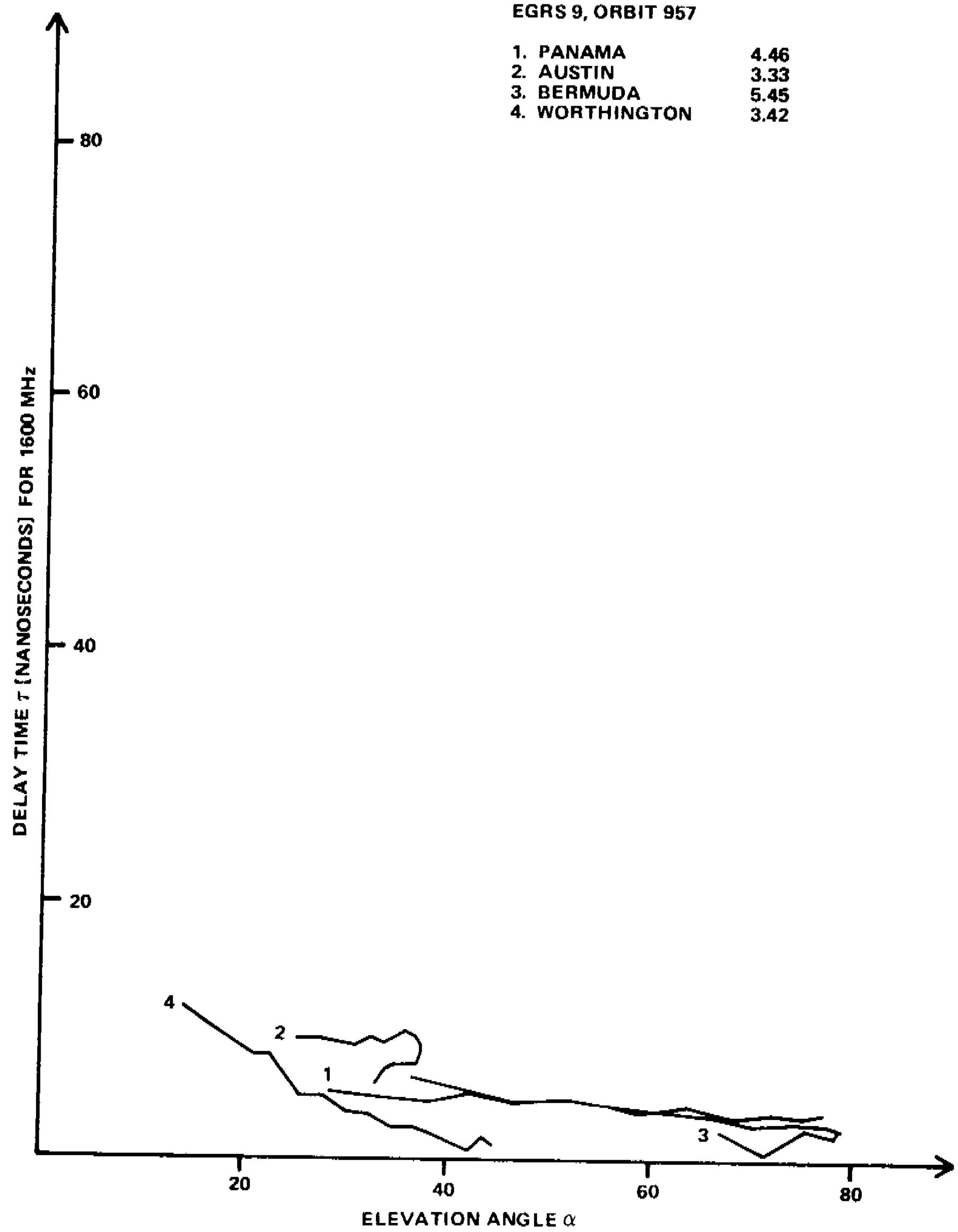

Figure 12. Results of an early morning four-station track.

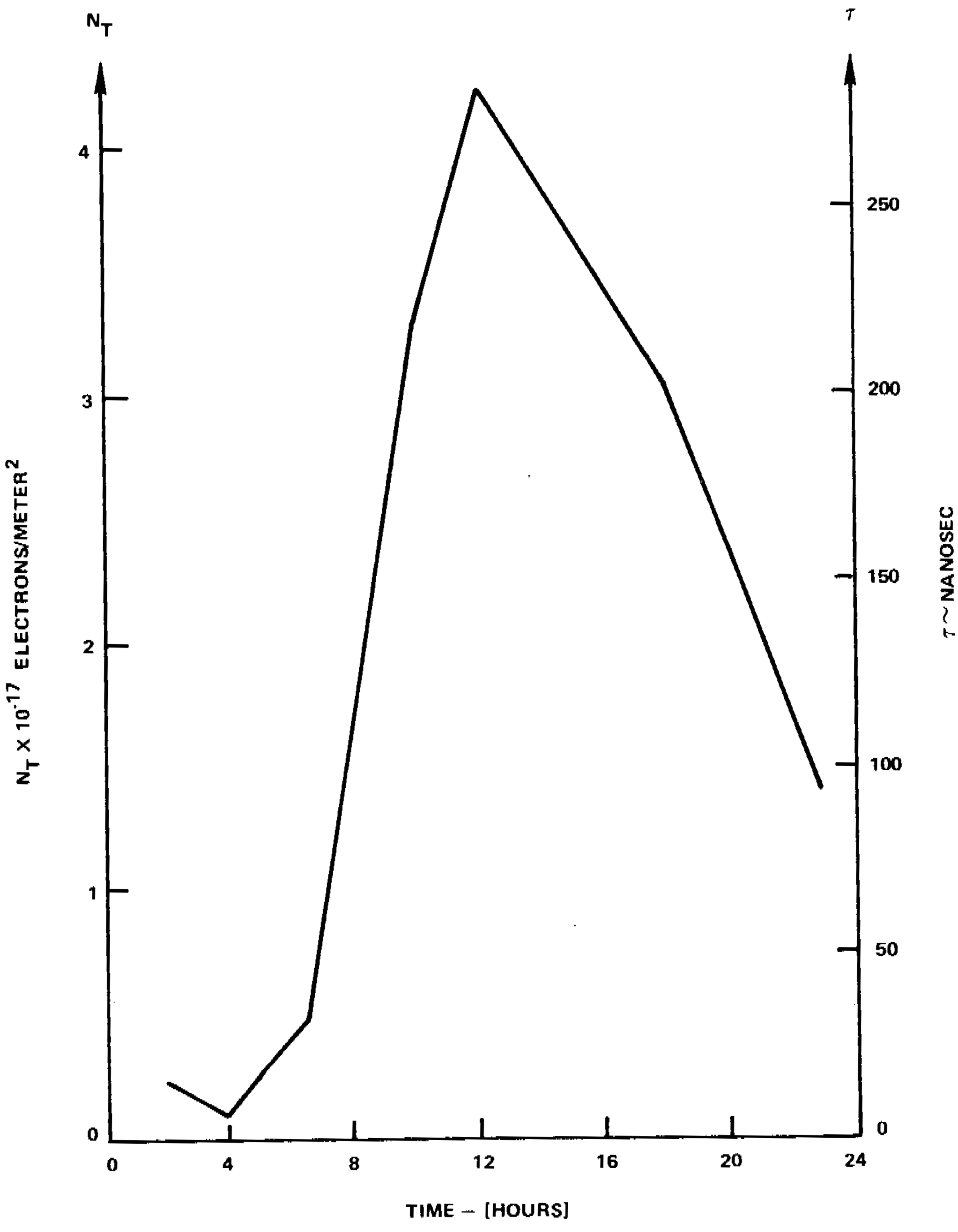

Figure 13. Results obtained from the station at Woleai.

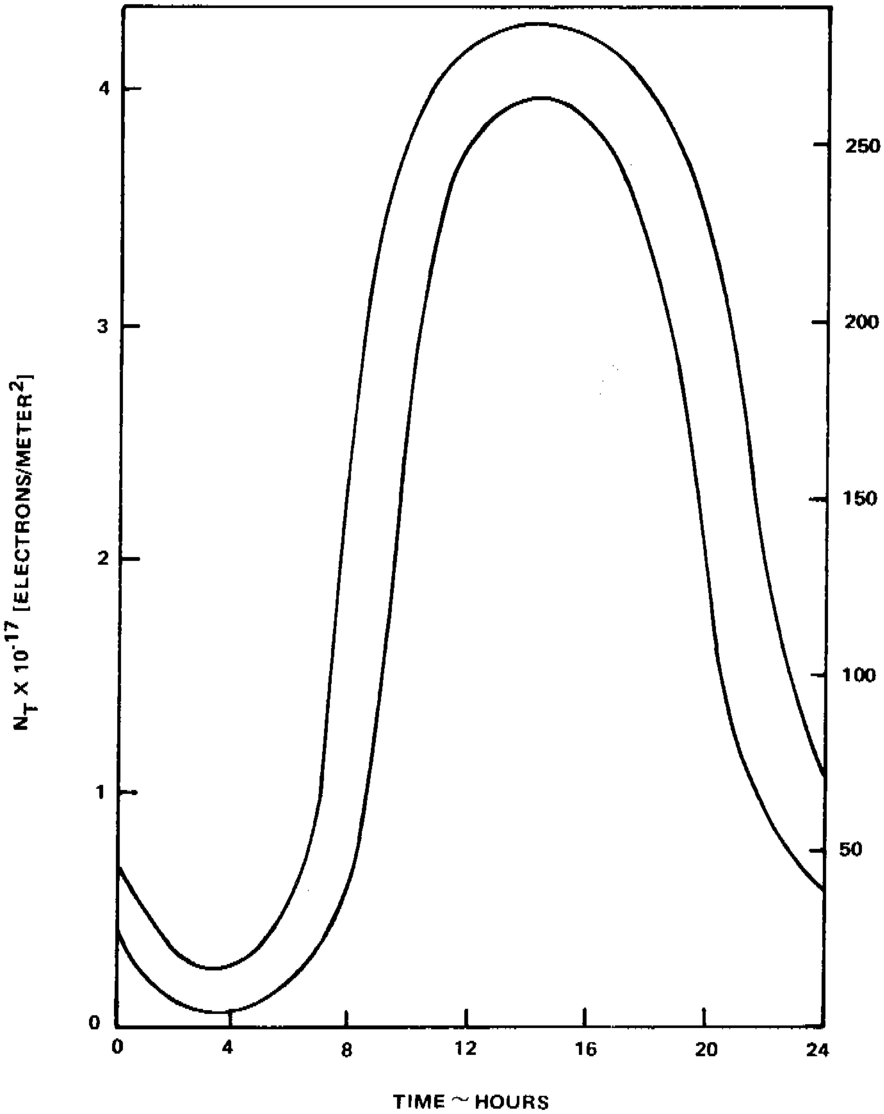

Figure 14. Diurnal region of vertical total electron count from about 70 measurements taken in the Pacific area between September 1965 and January 1966.

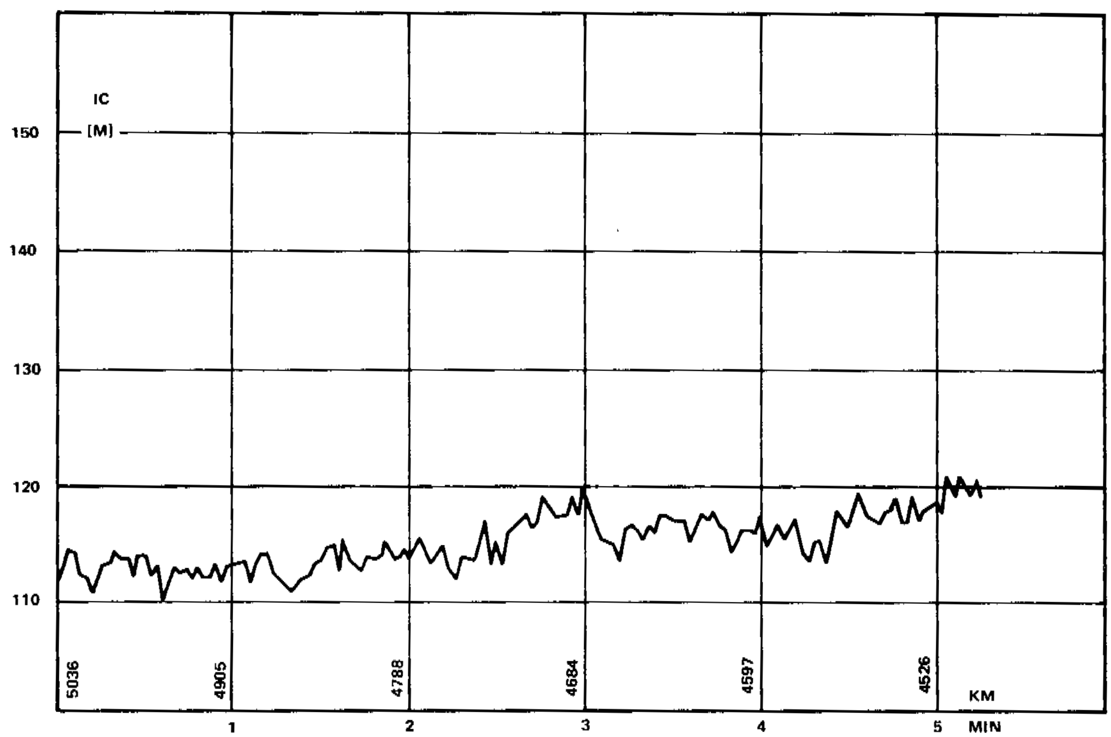

Figure 15. Typical example of noisiness of curves with respect to elevation angle, time of day, geographic location, and season.

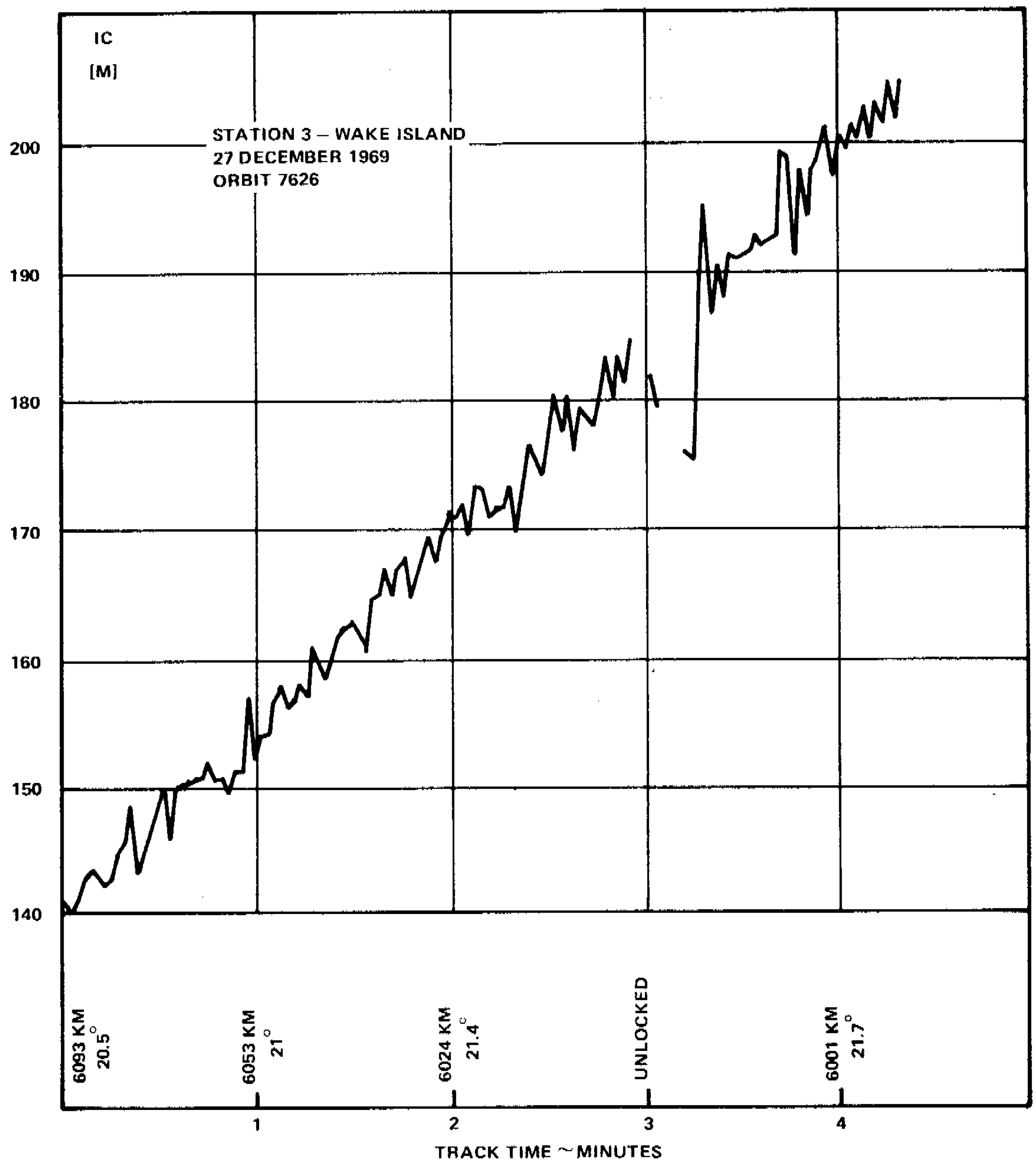

Figure 16. Another example of noisiness with same variable as those in Figure 15.

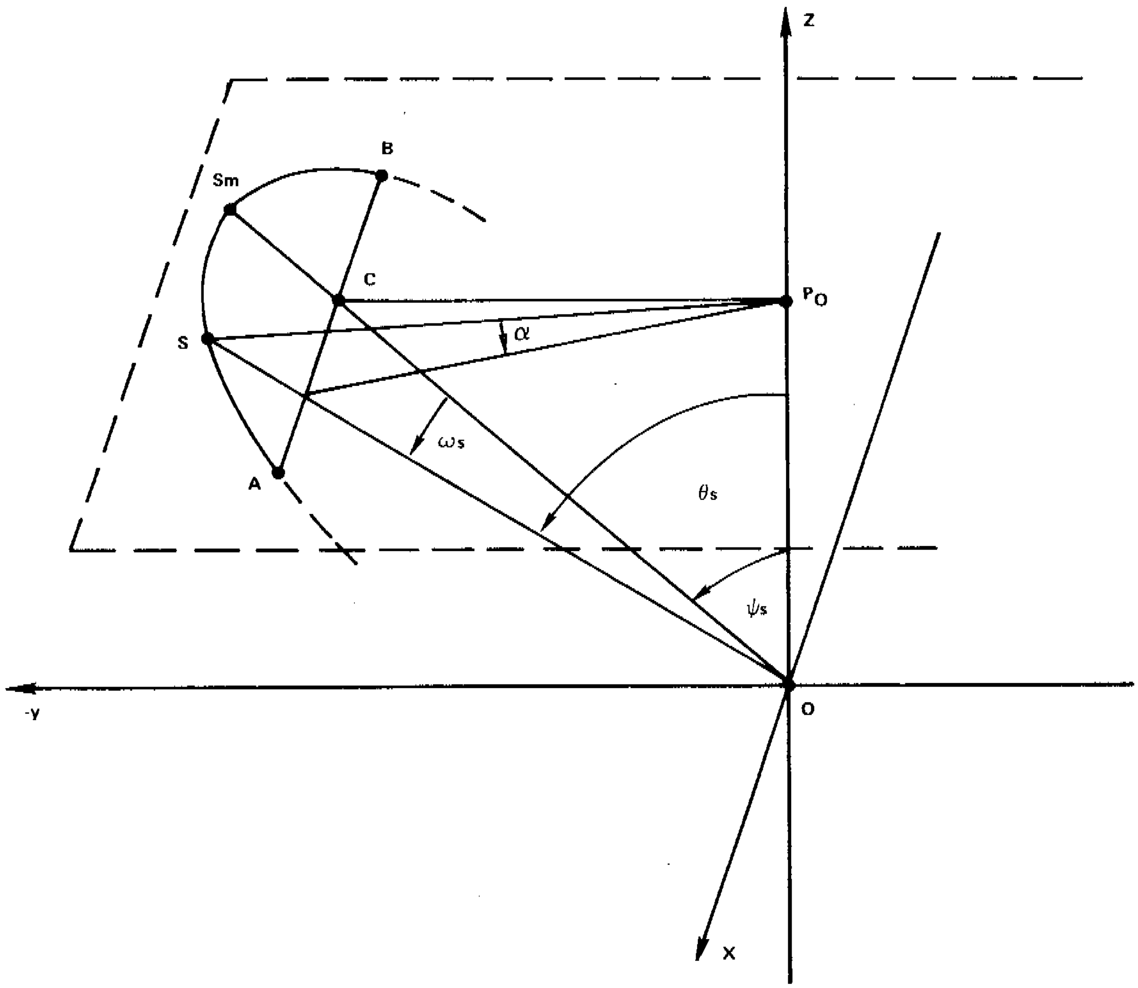

Figure 17. Plot of a satellite path.

I N P U T

RADIUS OF THE EARTH =6370. KM
DEL-ALPHA = 10. DEG   DEL-THETA = .005 DEG
ALPHA-MIN = 10. DEG   ALPHA-MAX = 70. DEG
H-LOW = 70. KM   H-HIGH =400. KM

F U N C T I O N S - U S E D

U = 1.0 - 0.10 * PSI

V=1.0-0.10 * OMEGA

(H1 H2 H3 HH1 HH2 HH3 A B C)
100. 120. 200. 140. 200. 250. .6 1.0 1.5

O U T P U T

	ALPHA (DEG)	ELECTRONS (E-17/M**2)
1	10.	.27
2	20.	.43
3	30.	.42
4	40.	.39
5	50.	.36
6	60.	.34
7	60.	.40
8	50.	.47
9	40.	.59
10	30.	.74
11	20.	1.19
12	10.	1.90

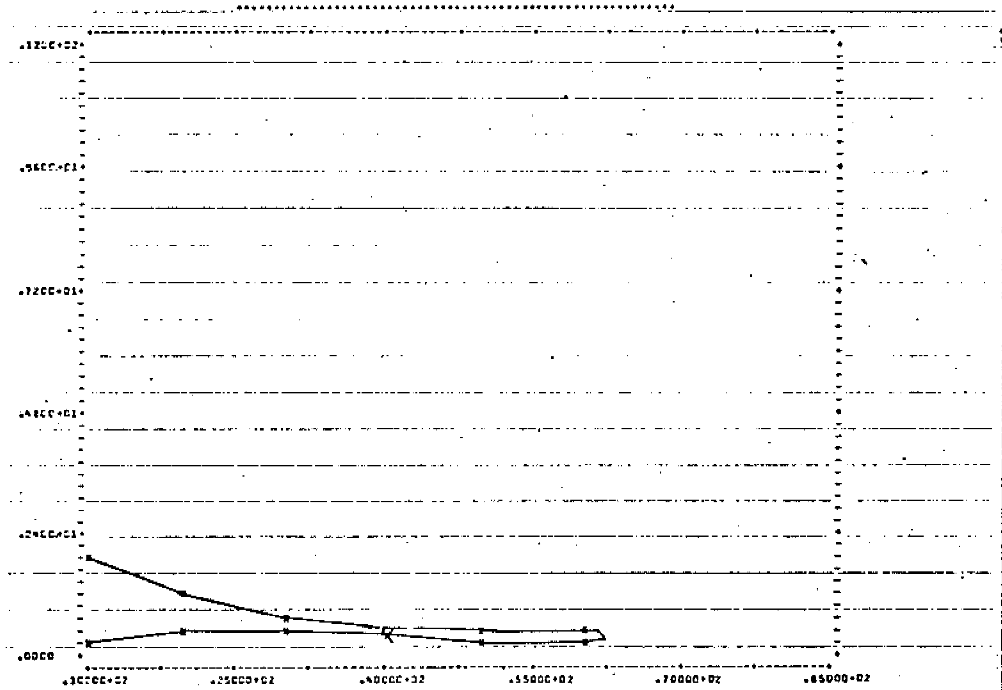

Figure 18(a). Total electron content calculated along the direction alpha (Figure 17).

I N P U T

RADIUS OF THE EARTH =6370. KM

DEL-ALPHA = 10. DEG	DEL-THETA = .005 DEG
ALPHA-MIN = 10. DEG	ALPHA-MAX = 70. DEG
H-LOW = 70. KM	H-HIGH =400. KM

F U N C T I O N S - U S E D

W = 1.0 - 0.10 * PSI

V = 1.0 - 0.05 * OMEGA

W (H1	H2	H3	HH1	HH2	HH3	A	B	C)
70.	95.	210.	120.	215.	305.	1.6	4.0	6.0

O U T P U T

	ALPHA (DEG)	ELECTRONS (E-17/M**2)
1	10.	3.84
2	20.	3.62
3	30.	3.11
4	40.	2.71
5	50.	2.43
6	70.	2.26
7	60.	2.45
8	50.	2.86
9	40.	3.45
10	30.	4.42
11	20.	6.14
12	10.	9.35

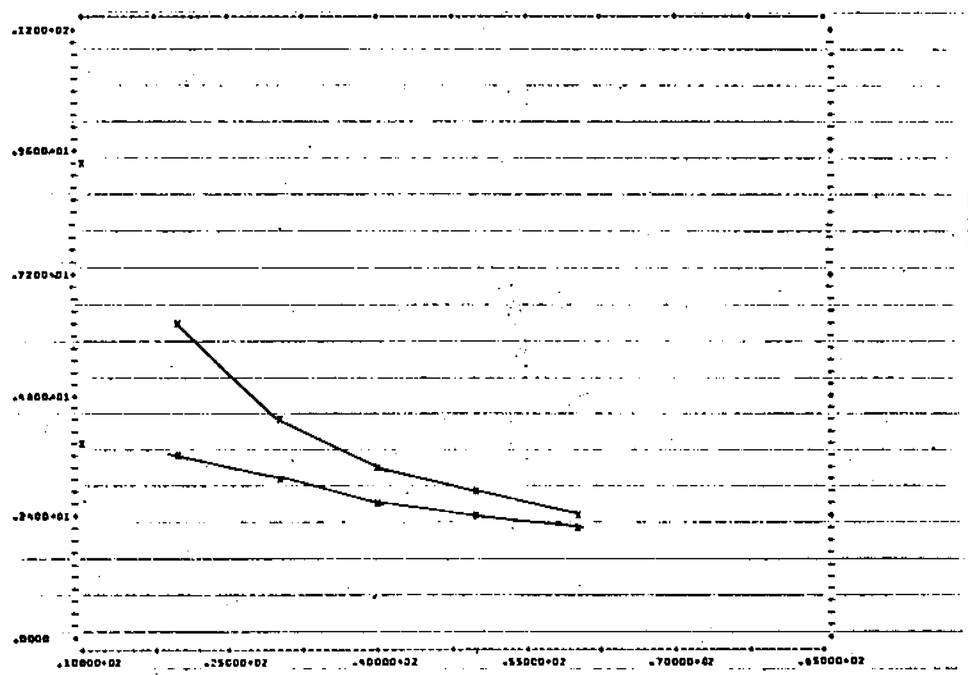

Figure 18(b). Total electron content calculated along the direction alpha (Figure 17).

I N P U T

RADIUS OF THE EARTH =6370. KM
DEL-ALPHA = 10. DEG   DEL-THETA = .005 DEG
ALPHA-MIN = 10. DEG   ALPHA-MAX = 70. DEG
H-LOW = 70. KM   H-HIGH =400. KM

F U N C T I O N S - U S E D

U = 1.0 - 0.10 * PSI

V = 1.0 - SIN(15*OMEGA-20*DELTA)

W (H1	H2	H3	HH1	HH2	HH3	A	B	C)
70.	95.	210.	120.	215.	305.	1.6	4.0	6.0

O U T P U T

	ALPHA (DEG)	ELECTRONS (E-17/M**2)
1	10.	.58
2	20.	.99
3	30.	1.43
4	40.	2.75
5	50.	2.45
6	60.	2.C0
7	60.	3.71
8	50.	4.48
9	40.	5.59
10	30.	7.25
11	20.	9.16
12	10.	9.87

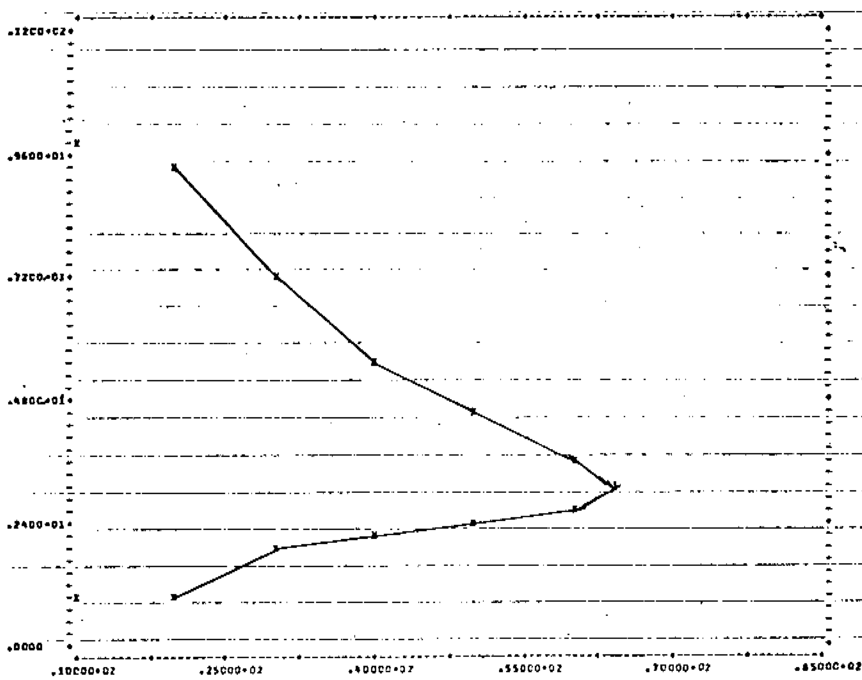

Figure 18(c). Total electron content calculated along the direction alpha (Figure 17).

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I N P U T

RADIUS OF THE EARTH =6370. KM
DEL-ALPHA =  3. DEG          DEL-THETA = .005 DEG
ALPHA-MIN =  9. DEG          ALPHA-MAX = 30. DEG
H-LOW = 70. KM               H-HIGH =400. KM

F U N C T I O N S - U S E D

W = 1.0 - 0.10 * PSI

V = 1.0 - 0.05 * OMEGA

W (H1   H2   H3   HH1  HH2  HH3    A    B    C)
   70.  90. 210. 120. 215. 305.  1.6  4.0  6.0

O U T P U T
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	ALPHA (DEG)	ELECTRONS (E-17/M**2)
1	9.	2.88
2	12.	2.95
3	15.	2.72
4	18.	2.99
5	21.	2.75
6	24.	2.66
7	27.	2.59
8	27.	3.07
9	24.	3.46
10	21.	3.90
11	18.	4.47
12	15.	5.05
13	12.	5.81
14	9.	6.69

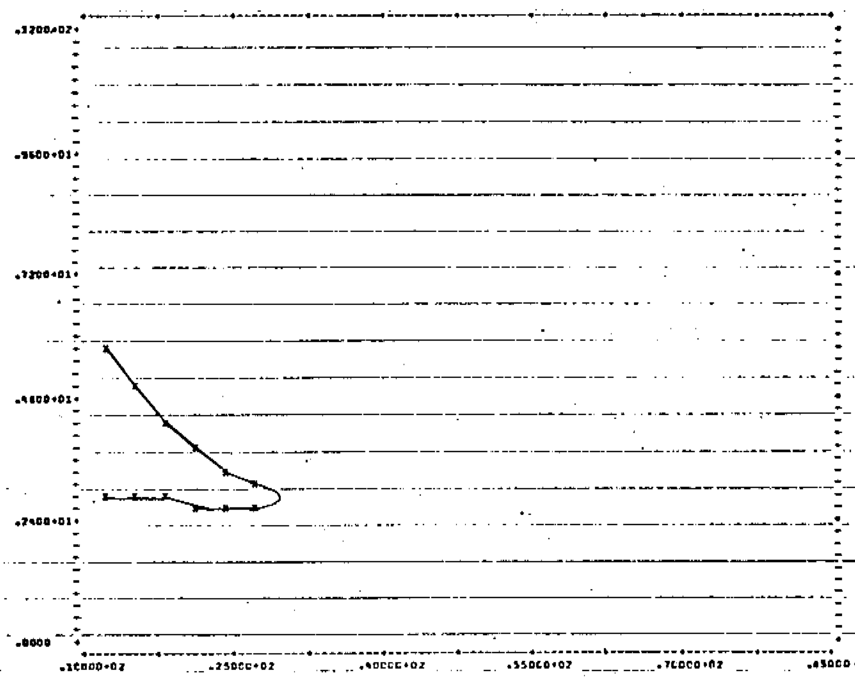

Figure 18(d). Total electron content calculated along the direction alpha (Figure 17).

I N P U T

RADIUS OF THE EARTH =6370. KM
DEL-ALPHA = 3. DEG   DEL-THETA = .005 DEG
ALPHA-MIN = 9. DEG   ALPHA-MAX = 30. DEG
H-LOW = 70. KM   H-HIGH =4CO. KM

F U N C T I O N S - U S E D

U = 1.0 - 0.01 * PSI

V = 1.0 - SIN(15*OMEGA-90*DELTA)

W (H1 H2 H3 HH1 HH2 HH3 A B C)
70. 85. 210. 120. 215. 305. 1.6 4.0 6.0

O U T P U T

	ALPHA (DEG)	ELECTRONS (E-17/M**2)
1	9.	.74
2	12.	1.27
3	15.	2.10
4	18.	3.02
5	21.	3.83
6	24.	4.54
7	27.	5.21
8	27.	6.10
9	24.	9.02
10	21.	9.40
11	18.	10.40
12	15.	10.65
13	12.	10.27
14	9.	8.95

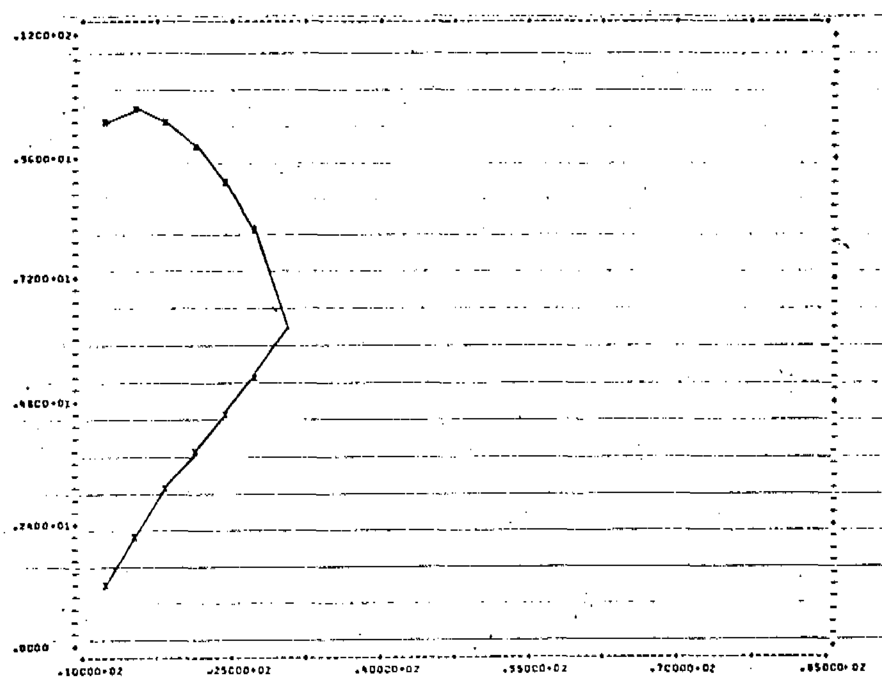

Figure 18(e). Total electron content calculated along the direction alpha (Figure 17).